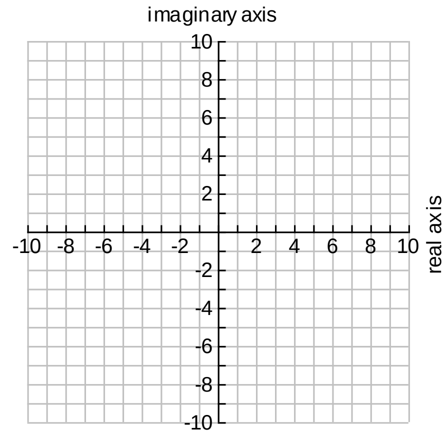
9.4 Trigonometric Form of Complex Numbers Notes



The complex number $a+bi$ can be thought of as an ordered pair (a,b). We graph it on the **complex plane** where the horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis.**

Absolute Value or Modulus: $|a+bi|=\sqrt{a^2+b^2}$. (The distance between the number and the origin on the complex plane.)

Examples: Graph each complex number and find its absolute value.

a) $5-i$ b) $-6+2i$

Trigonometric Form of a Complex Number

If $z=a+bi$ is a complex number, then the trigonometric form of z is

$$z=r(\cos\theta+i\sin\theta), \text{ sometimes abbreviated } z=r\,\text{cis}\,\theta,$$

where r is called the **modulus** and θ is called the **argument,** defined as the angle in standard position whose terminal side contains the point (a,b).

$$r=\sqrt{a^2+b^2}$$
$$a=r\cos\theta \text{ and } b=r\sin\theta.$$

$$\theta=\tan^{-1}\left(\frac{b}{a}\right)$$

We usually use the smallest possible nonnegative angle for θ.

Examples: Write each complex number in trigonometric form. Express θ in degrees.

a) $-2\sqrt{3}+2i$ b) $5-4i$

Example: Write the complex number $12\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ in the form $a+bi$.

Product and Quotient of Complex Numbers

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$$z_1 z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$$

Examples: Find the product and quotient using trigonometric form.

$$z_1 = 4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right),\quad z_2 = 8\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$$

a) Find $z_1 z_2$

b) Find $\frac{z_1}{z_2}$

Complex Conjugates

The conjugate of $r(\cos(\theta)+i\sin(\theta))$ is $r(\cos(-\theta)+i\sin(-\theta))$

A complex number times its conjugate equals r^2.

Proof: $r(\cos\theta+i\sin\theta)\cdot r(\cos(-\theta)+i\sin(-\theta))$

$$= r^2(\cos(\theta-\theta)+i\sin(\theta-\theta))$$

$$= r^2(\cos 0 + i\sin 0)$$

$$= r^2(1+0i) = r^2$$

Example: Find the product of the following and its conjugate: $6\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$.